**Write up on calculating degree exponent**

The following fitting procedure has been described in detail in reference [1]. We start with a set of indegrees for N supplier nodes in the supply network of a focal company. So, for each supplier the indegree is denoted by . From this set of indegrees, we calculate the probability of a supplier node having indegree . This is done by counting the number of nodes with indegree and divide by total number of nodes in the network. When calculated for all ranging from maximum () to minimum (), describes the empirical indegree probability distribution. An indegree probability distribution that follows power-law has the form,

where denotes the indegree and is called degree exponent. By fitting the empirical indegree probability distribution to the above describe formula, we aim to estimate degree exponent [2]. The data fitting process is done in following steps:

1. Choose a value between Next, we estimate a value for degree exponent corresponding to this value of ;
2. For the pair of ) parameters, the indegree probability distribution is given by

and, it’s associated cumulative distribution function (CDF) is given by

Note: is called Hurwitz zeta function [3] which is a generalization of the Riemann zeta function, so it is also known as the generalized zeta function. It is defined by the formula

1. Next, we determine the maximum distance between the CDF of the real data and the fitted CDF . This in literature is called Kormogorov-Smirnov test,
2. We calculate the maximum distance for all possible values of between maximum indegree () and minimum indegree (). We find the for which parameter minimize and then fix that value as the small-indegree cut-off on the fitted indegree distribution.
3. Finally, we calculate the degree exponent as described in Step 1. using that minimize . Further, the standard error for the obtained degree exponent is,

As an example, we present result for 5-Tier supply network for “Company X” FactSet ID: ’1228’ for the Quarter- 1 [April 3rd, 2003 - July 1st, 2003]. The number of supplier nodes is N=3967 and L= 25264 number of links between the nodes. We only consider indegree which is the number of links coming into a node. Based on the fitting procedure we found the maximum distance is minimized for and the degree exponent is 2.872 with standard error, .

**Reference**

[1]Barabási, A. L. (2016). The scale-free property. *Network science*. Cambridge university press.

[2] A. Clauset, C.R. Shalizi, and M.E.J. Newman. Power-law distributions in empirical data. SIAM Review S1: 661-703, 2009.

[3] Weisstein, Eric W. "Hurwitz Zeta Function." Wolfram Research, Inc. (2002).